Neutral-Pion Two-Photon Decays from Lattice QCD

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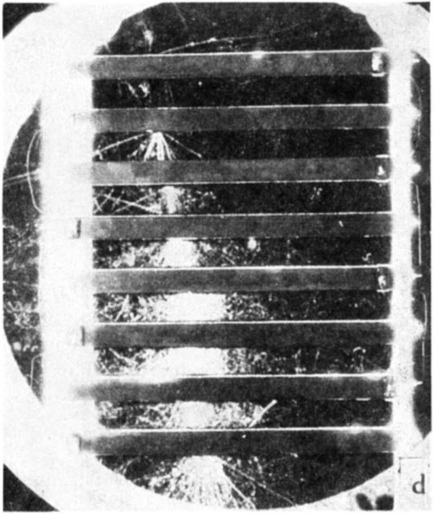
The Neutral Pion

- First detected in cosmic-ray showers `PR73,41 (1948)`
- Produced in Berkeley cyclotron `PR78,802 (1950)`
- Lightest of all hadronic states
- Yet difficult to investigate with precision experiments due to neutral charge of itself and its decay products

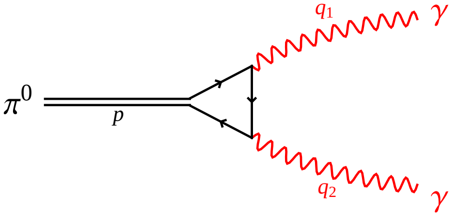


Goldstone Two-Photon Decay

Anomalous Symmetry Breaking

- Bell and Jackiw (1969) discovered that chiral symmetry is anomalously broken by the regulator of the field theory; result extended to all orders by Adler

- $\Gamma(\pi^0 \to \gamma\gamma) \approx \frac{N_c^2 M_\pi^3 \alpha^2}{144\pi^3 F_\pi^2}$
- Resolved an factor of 1000 error in theory prediction



Goldstone Two-Photon Decay

Chiral Perturbation Theory

- Non-zero light-quark masses: to NLO in XPT
- Isospin breaking: Mixing with η and η'
- Electromagnetic corrections

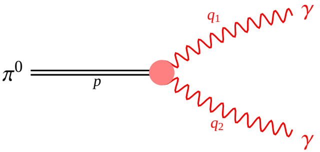
$$
\begin{aligned}
\Gamma_{\pi^0\to\gamma\gamma} &= 7.74 \text{ eV} \\
&\downarrow \quad \text{NLO} + \text{mixing} + \text{EM} \\
\Gamma_{\pi^0\to\gamma\gamma} &= 8.08(10) \text{ eV}
\end{aligned}
$$

What Makes This Quantity So Interesting?

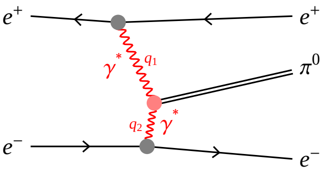
- Directly tests one of the most striking predictions of the Standard Model: anomalous symmetry breaking
- Pion is probably well described by the anomaly + XPT (2% uncertainties), but heavier states are probably not
- Error in determination of the light-quark mass ratio is dominated by uncertainty in $\Gamma(\eta \to \gamma\gamma)$
- Dominant contribution to hadronic light-by-light in muon $g - 2$

Two-Photon Processes

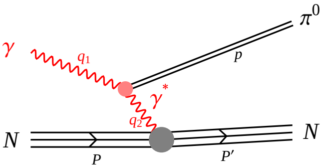
- $\pi \to \gamma\gamma$: Neutral meson decay (e.g. CERN SPS)



- $\gamma^*\gamma^* \to \pi$: Photon fusion (e.g. CELLO, CLEO, BaBar)



- $\gamma^*\gamma \to \pi$: Primakoff effect; photoproduction (e.g. PrimEx, GlueX)



Two-Photon Decay

- Simple, direct method
- Measure positron counts as a function of distance between plates
- $\Gamma_{\pi^0 \to \gamma\gamma} = 7.34(18)(11)$ eV (CERN 1985)
- Only gives $F_{\pi\gamma\gamma}(Q_1^2 = 0, Q_2^2 = 0)$ (on-shell photons)

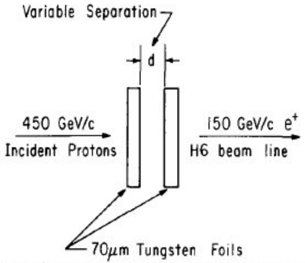
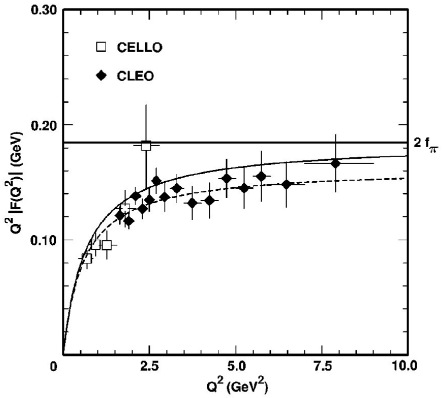


Fig. 1. Schematic arrangement for the measurementoftheπ^0 lifetime.

Photon Fusion

- Uses electron-positron collisions to produce neutral mesons
- $\Gamma_{\pi^0\to\gamma\gamma} = 7.70(50)(50)$ eV (Crystal Ball 1988)
- Allows off-shell exploration, but CELLO (1991) and CLEO (1998) only measure Q^2 dependence and do not get the decay width
- BaBar new high-Q^2 data defies earlier model expectations



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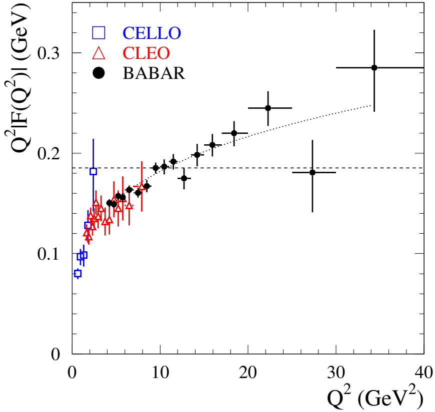
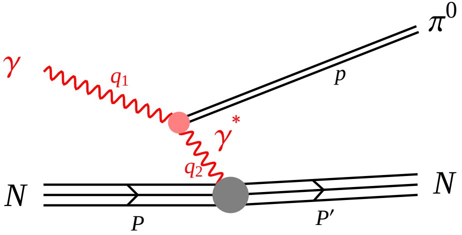


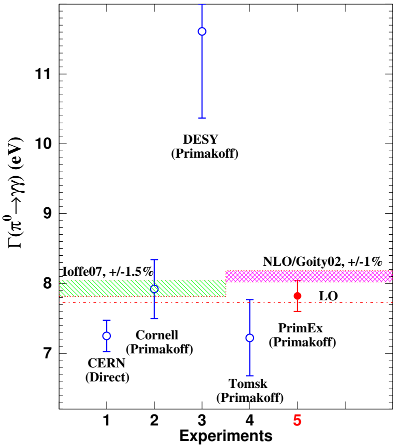
Primakoff Effect

- Photon beam scattering on nuclear Coulomb potential
- $\Gamma_{\pi^0\to\gamma\gamma} = 8.02(42)$ eV (Cornell 1974)
- $\Gamma_{\pi^0\to\gamma\gamma} = 11.75(126)$ eV (DESY 1970)
- $\Gamma_{\pi^0\to\gamma\gamma} = 7.31(55)$ eV (Tomsk 1970)



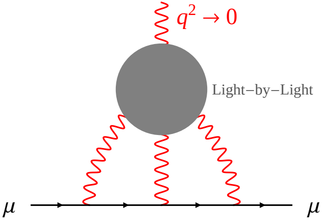
PrimEx

- Precision measurement needed to resolve discrepancies
- PrimEx currently measuring π, η and η' form factors
- $\Gamma_{\pi^0\to\gamma\gamma} = 7.82(14)(17)$ eV (PrimEx 2010)

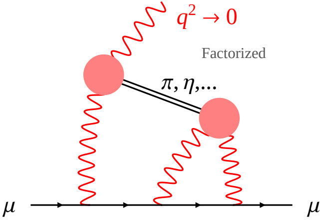


Four-Photon Process

- $\gamma\gamma \to \gamma\gamma$: Light-by-Light



- $\gamma\gamma \to \pi, \eta, \ldots \to \gamma\gamma$: Leading hadronic light-by-light



QED Eigenstates without QED

- Since the photon is not an eigenstate of QCD, standard lattice techniques will fail; a 1^{--} interpolating operator yields ρ (or $\pi\pi$) rather than γ.
- An elegant solution is provided by Ji & Jung PRL86, 208. We want this matrix element:

$$\langle \gamma(q_1, \lambda_1)\gamma(q_2, \lambda_2)|\Phi(p)\rangle$$

- Perform a Lehmann-Symanzik-Zimmermann (LSZ) reduction:

$$-\lim_{q' \to q} \epsilon_\mu^{(1)*}\epsilon_\nu^{(2)*} {q_1'}^2 {q_2'}^2 \int d^4x\, d^4y\, e^{iq_1'\cdot y + iq_2'\cdot x}\langle 0|T\{A^\mu(y)A^\nu(x)\}|\Phi(p)\rangle$$

Perturbative QED

- Although we cannot treat the A fields in QCD, we can use perturbative QED to integrate them out:

$$\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\; e^{iS_{\mathrm{QED}}} A^{\mu}(y)A^{\nu}(x) \approx$$

$$\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\; e^{iS_0} \left(\ldots + \left[\bar{\psi}\gamma^{\rho}\psi A_{\rho}\right](z)\left[\bar{\psi}\gamma^{\sigma}\psi A_{\sigma}\right](w) + \ldots\right) A^{\mu}(y)A^{\nu}(x)$$

- Then we Wick contract the photon fields into propagators

$$-e^2 \lim_{q'\to q} \epsilon_{\mu}^{(1)*}\epsilon_{\mu}^{(2)*} {q_1'}^2 {q_2'}^2 \times$$

$$\int d^4x\, d^4y\, d^4w\, d^4z\, e^{iq_1'\cdot x} D^{\mu\rho}(0,z) D^{\nu\sigma}(x,w) \langle 0| T\{j_{\rho}(z) j_{\sigma}(w)\} |\Phi(p)\rangle$$

Into Euclidean Space

- Using the explicit form of the photon propagator, most of these integrals go to delta functions:

$$e^2 \epsilon_\mu^{(1)*} \epsilon_\mu^{(2)*} \int d^4x \, e^{iq_1 \cdot y} \langle 0 | T\{j^\mu(0) j^\nu(y)\} | \Phi(p) \rangle$$

- This, we can rotate into Euclidean space unless we hit a pole; we must keep $q^2 < M_\rho^2$ (or $E_{\pi\pi}^2$).

$$\frac{e^2 \epsilon_\mu^{(1)} \epsilon_\mu^{(2)}}{\frac{Z_\Phi(p)}{2E_\Phi(p)} e^{-E_\Phi(p)(t_f - t)}} \int dt_i \, e^{-\omega_1 (t_i - t)} \times$$
$$\left\langle T \left\{ \int d^3\vec{x} \, e^{-i\vec{p}\cdot\vec{x}} \varphi_\Phi(\vec{x}, t_f) \int d^3\vec{y} \, e^{i\vec{q}_2 \cdot \vec{y}} j^\nu(\vec{y}, t) j^\mu(\vec{0}, t_i) \right\} \right\rangle$$

Lattice Three-Point Correlator

- This expression we can evaluate on the lattice.
  The term between the angled brackets is just the three-point function with an arbitrary meson on one end and vector currents at the other end and inserted.

$$\frac{e^2 \epsilon_\mu^{(1)} \epsilon_\mu^{(2)}}{\frac{Z_\Phi(p)}{2E_\Phi(p)} e^{-E_\Phi(p)(t_f - t)}} \int dt_i \, e^{-\omega_1 (t_i - t)} \times$$

$$\left\langle T \left\{ \int d^3\vec{x} \, e^{-i\vec{p}\cdot\vec{x}} \varphi_\Phi(\vec{x}, t_f) \int d^3\vec{y} \, e^{i\vec{q}_2\cdot\vec{y}} j^\nu(\vec{y}, t) j^\mu(\vec{0}, t_i) \right\} \right\rangle$$

- The remaining parts describe how to combine QCD states into a photon of the appropriate energy.
- The most straightforward way to evaluate this is to compute the three-point function on all t_i and perform the integral explicitly.

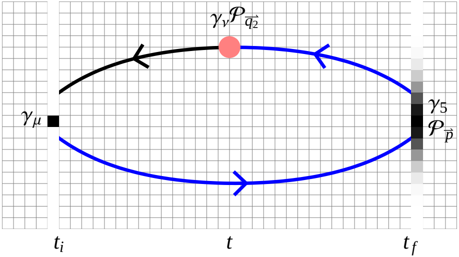
Lattice Setup

Action and Parameters

- HSC's anisotropic $20^3 \times 128$ $N_f = 2+1$ clover lattices
- Clover action is inexpensive and provides $O(a)$ improvement
- Fine temporal spacing provides excellent sampling of the integrand
- $a_s \approx 0.12$ fm, $M_\pi \in \{830, 560, 450, 390\}$ MeV

Lattice Setup

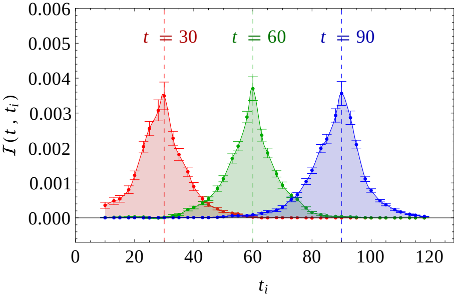
Three-Point Correlator



- Meson location fixed: $t_f = 120$, $\vec{p} = \{0, 0, 0\}$, Gaussian-smeared
- Sequential source from point-source at t_i, continuing through t_f
- Momentum projection $0 \leq |\vec{q_2}|^2 \leq 5$ at t

Integrand Evaluation

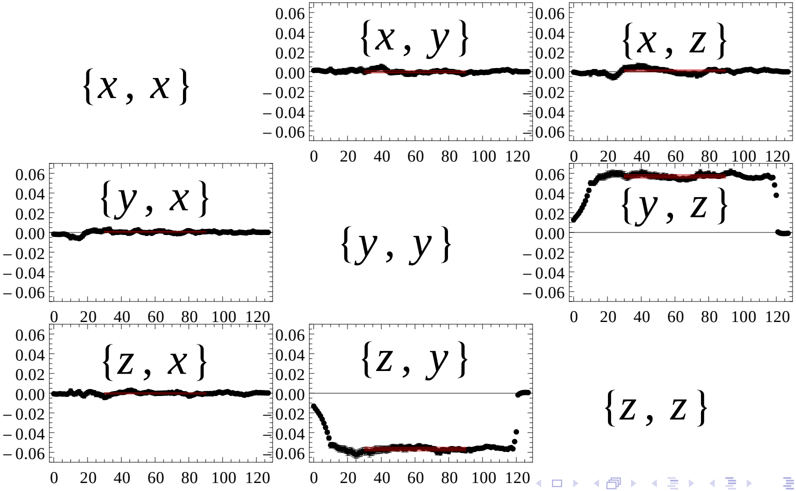
$$\frac{Z_\Phi(p)}{2E_\Phi(p)} \frac{e^{-\omega_1(t_i-t)}}{e^{-E_\Phi(p)(t_f-t)}} \mathcal{C}_{PVV}(t_f, t, t_i)$$



- Check width: If too narrow, cannot integrate accurately. If too wide, cannot find a plateau.
- Check distortion: Due to Dirichlet BCs and sink location.

The Integral

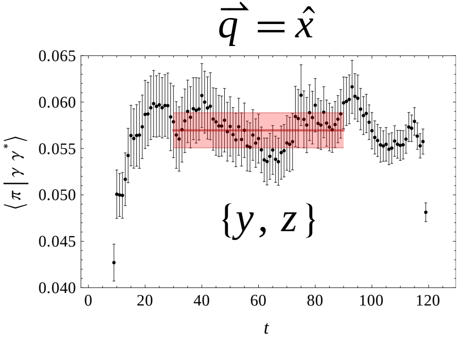
$$\vec{q} = \hat{x}$$



The Integral

A Non-Zero Element

- Integral only nonzero when $\varepsilon_{\mu\nu\rho\sigma}\epsilon^{\mu}\epsilon^{\nu}q_1^{\rho}q_2^{\sigma} \neq 0$
- We see a clear plateau in the expected region, away from $t = 0$ and $t = t_f$
- There may be exponential contamination due to excited states. Not seen here since the gap is large?

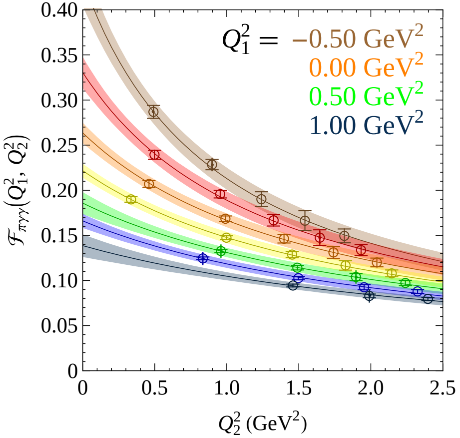


$\mathcal{F}_{\pi\gamma\gamma}(Q_1^2, Q_2^2)$

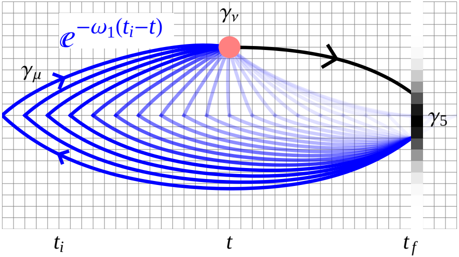
Monopole Fit

- We can set Q_1^2 arbitrarily. Useful for photon fusion?
- The data are well described by a monopole fit:

$$\mathcal{F}(Q_1^2, Q_2^2) = \frac{F(Q_1^2)}{1 + Q_2^2/M_{\text{pole}}^2(Q_1^2)}$$



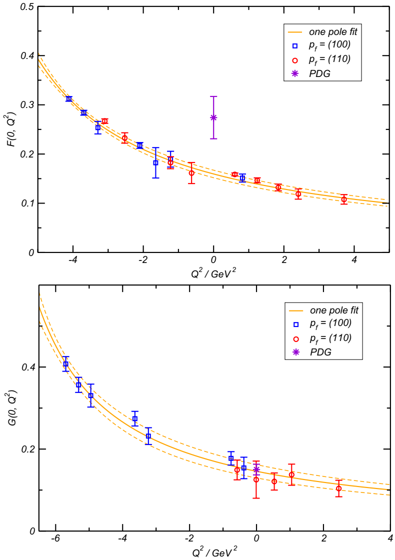
Fast Method



- Fold the exponential and integral over t_i into a sequential source
- Disadvantages: Cannot directly examine integrand; cannot vary Q_1^2 (without recalculating sequential propagator)
- Advantages: T times faster than the slow method

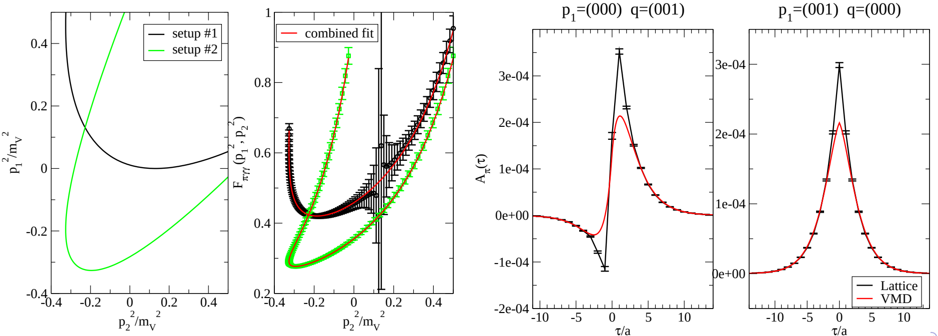
Previous Work on Charmonium

- Two-Photon Decays of Charmonia from Lattice QCD Dudek & Edwards, PRL97:172001, 2006
- Used clover fermions on quenched lattices with $a \approx 0.047$ fm



JLQCD's Work

- Two-Photon Decay of the Neutral Pion in Lattice QCD
  arXiv: 1206.1375
- Used 2+1-flavor frozen overlap with M_π 540 MeV to 290 MeV
- Two volumes: $16^3 \times 48$ and $24^3 \times 48$ at $a \approx 0.11$ fm
- Huge statistics: all-to-all propagators



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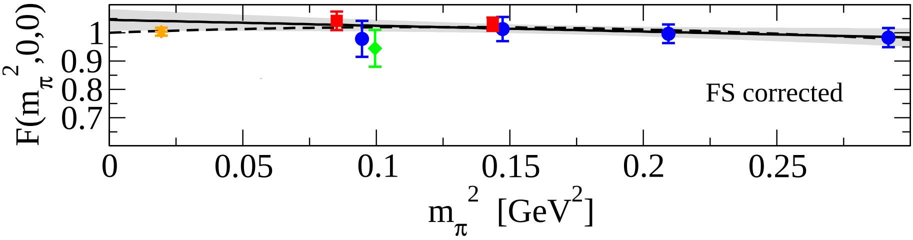

$$\Gamma_{\pi^0 \to \gamma\gamma} = 7.83(31)(49) \text{ eV}$$

Summary

- Conclusions
  - The method of Jung & Ji allows access to the two-photon decays of neutral mesons on the lattice
  - JLQCD sees nice agreement with PrimEx's recent pion width
- Future Work
  - Extrapolation to light quark masses
  - Fast method with conserved currents
  - Calculation of two-photon decays of η, scalar and axial mesons